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A meteor propagation model based on fitting the differential equations of meteor motion

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The differential equations that describe meteor motion through the atmosphere during the time of luminous flight does not have a closed-form solution to the state propagation vectors. Presented herein is a preliminary model that is an approximate parameterization to the integral solutions that are strictly dependent on the mass loss parameter β. The resultant model for position, and thus velocity, as a function of time and β, can be used to fit meteor and fireball trajectories that show deceleration over the entire visible duration of the flight profile.

1 Introduction

The deceleration of a meteor through the atmosphere has been discussed in a number of papers that describe the classical physical behavior through a set of dynamical equations of motion (e.g., Pecina and Ceplecha, 1984; Gritsevich, 2009; Gritsevich and Koschny, 2011). The resultant solution for the velocity propagation, however, has only been derived down to a complicated expression for the differential equation of velocity with respect to time. The integral equation thus obtained from the $\mathrm{d}V/\mathrm{d}t$ expression has not been shown to have a closed form solution. Thus to fit the state vector measurements of a decelerating meteor, one has to resort to iteratively solving the differential equation or use a simpler model for velocity, such as one that is constant $V = V_0$ or exponential in time $V = V_0 - Ck\exp(kt)$ (Whipple and Jacchia, 1957). Since these simpler models may not be valid over the entire duration of a deeply penetrating fireball, meteorists often resort to fitting them over shorter time segments and linking the solutions together. Thus, it would be desirable to find a more general expression for the propagation that would permit a single fit along the entire luminous flight path. This can also be used as a meteor propagation model for fully-coupled multi-camera trajectory estimation (Gural, 2012).

2 The integral equation for time and velocity

The basic differential equations for drag and mass loss will not be repeated here, but can be found in the paper by Gritsevich (2009), and the nomenclature follows that paper's convention. Under the assumptions of no deflection from straight line path, isothermal atmosphere, and power-law relationship between shape and mass, that paper derives the differential equations for mass-versus-height and velocity-versus-height. The resultant equations are found to be dependent on only three dimensionless parameters: the ballistic coefficient α, the mass loss parameter β, and the shape-to-mass power exponent μ. Furthermore, a differential equation for velocity $\mathrm{d}V/\mathrm{d}t$ was also derived, that is dependent on the entry velocity V_e, the trajectory angle γ, the atmospheric scale height h_0, and the mass loss parameter β, as shown in equation (1), where $\nu = V(t)/V_\mathrm{e}$ is the normalized velocity and $\mathrm{Ei}(y)$ is the exponential integral[1].

$$\frac{\mathrm{d}V}{\mathrm{d}t} = V_\mathrm{e}^2 \nu^2 \sin\gamma \, \frac{\mathrm{Ei}(\beta) - \mathrm{Ei}(\beta\nu^2)}{2h_0 e^{\beta\nu^2}} \,. \qquad (1)$$

The first three parameters mentioned represent a scale factor on the deceleration, and only the β parameter influences the shape of the velocity profile versus time. This inspired an intriguing thought that a general model for velocity could be obtained whose profile would only be a function of time t and β. The differential equation for velocity, when reformulated into an integral as in equation (2), relates a velocity ν_k to a time t_k. Unfortunately, this integral does not have a known closed-form solution. Note that the integral's limits are 1 to ν_k, representing times of 0 and t_k, respectively.

$$t_k = -\frac{2h_0}{V_\mathrm{e}\sin\gamma} \int \frac{e^{\beta\nu^2}}{\mathrm{Ei}(\beta) - \mathrm{Ei}(\beta\nu^2)} \, \frac{\mathrm{d}\nu}{\nu^2} \,. \qquad (2)$$

For a fixed value of β, one could perform the integration numerically for a sequence of integral limit values ν_k and build a curve of $\nu_k(t_k)$. If a functional form for the velocity profile could be obtained empirically, then, by parameterizing over the potential range of β values, the family of curves for $\nu(t, \beta)$ can be modeled, any one of which should match a given fireball's flight profile. To find a model for $\nu(t, \beta)$, the first step was to form the set of velocity curves using a Matlab script that was implemented to numerically integrate equation (2). The resulting velocity profiles as a function of time for various β values is shown in Figure 1, where the term outside the integral was set to unity. The mass loss parameter from previously measured meteors has been found to fall in a range of $\ln\beta$ between -2.5 and $+3.5$ based on prior results of dozens of deeply penetrating fireballs. Those were the limits used to build velocity curves with spacing between β values adjusted to span the shapes of the curves more uniformly.

[1] $\mathrm{Ei}(y) = \int_{-\infty}^{y} u^{-1}\exp(u)\mathrm{d}u$.

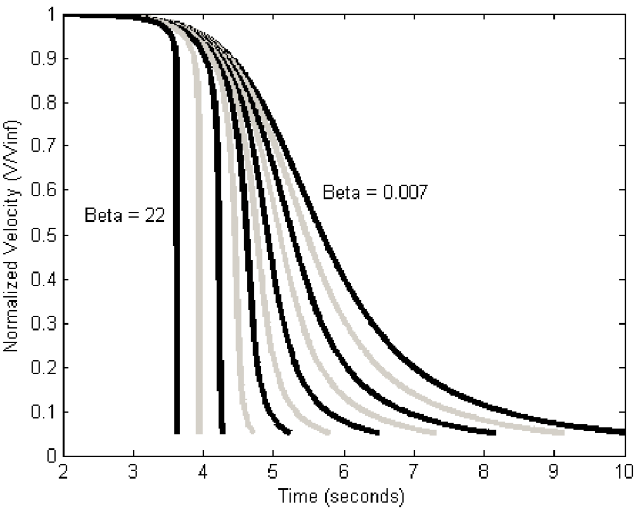

Figure 1 – Velocity versus time $\nu(t, \beta)$ profiles for various values of β (β = 0.0070, 0.7551, 1.5189, 2.2222, 2.9889, 3.7965, 4.6691, 5.8958, 8.1317, 12.6030, and 22.0027).

If an empirical formula for $\nu(t, \beta)$ could be determined, then the model could be used to minimize the error relative to actual velocity measurements and thus find the β value for a meteor. Alternatively, a formula for the position x as a function of time and β could also be empirically found; $x(t, \beta)$ and its derivative would then serve as the velocity model. Either model is useful because they both depend on a single parameter β and thus simplify the minimization of a cost function for doing a measurement to model fit. Since the model is most likely non-linear in nature, the single parameter minimization should also be more robust to potential issues with getting trapped in local minima. By integrating the individual velocity profiles from Figure 1, one obtains the position versus time curves shown in Figure 2 for the same set of β values.

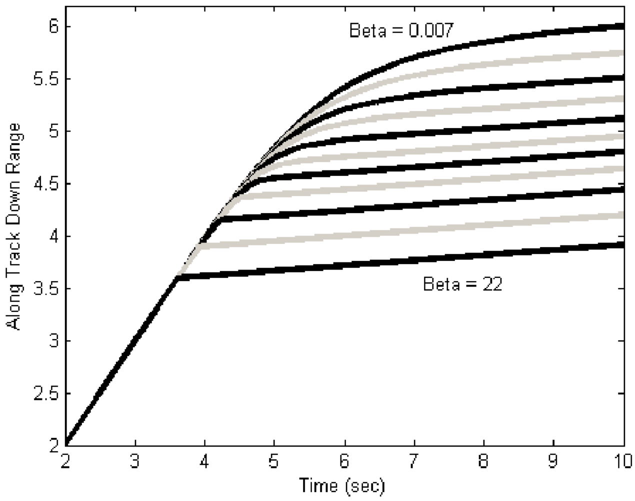

Figure 2 – Position versus time $x(t, \beta)$ profiles for various values of β (β = 0.0070, 0.7551, 1.5189, 2.2222, 2.9889, 3.7965, 4.6691, 5.8958, 8.1317, 12.6030, and 22.0027).

The position profiles show a slow change in time for small values of the mass loss parameter β and an increasing abrupt and discontinuous break as the mass loss parameter grows to greater than 5. These high β cases correspond to fireballs that catastrophically disintegrate as if they have slammed into a brick wall and very rapidly reached terminal velocity. But they also represent the most difficult to build a general propagation model for, because of the abrupt change in velocity. Continuous functions do not behave this way and are associated with ringing artifacts when trying to fit to sharp corners in data. At a minimum, the desire will be to find a monotonic and single-valued function with time.

3 Finding a model for position versus time

The simple expressions for meteor propagation, be it linear $x = V_0 t$ or with an exponential term $x = V_0 t - C \exp(kt)$, are not descriptive of the curves shown in Figure 2 for long-duration or significantly decelerating meteors (high β values). Since the integral equation is intractable, one could try to guess at a reasonable formula, but this approach too was found to be nearly impossible.

However, it turns out there is a software tool on the web for discovering underlying mathematical expressions in data. It is called EUREQA FORMULIZE for version II (Schmidt and Lipson, 2009) and is billed as *a software tool for detecting equations and hidden mathematical relationships in your data.* The software attempts many permutations of mathematical combinations of basic functions and builds on formulae with the most promising residuals relative to the measurements. It combines, trims, and mutates function combinations, ultimately trying to find the simplest mathematical formula which could describe the underlying data.

The FORMULIZE software was attempted on the positional data curves $x(t, \beta)$ obtained by integrating the velocity profiles rather than applying FORMULIZE directly to the velocity profiles $\nu(t, \beta)$ themselves. This was preferred, since taking the derivative of $x(t, \beta)$ later was deemed simpler for formulating an analytic velocity model after the positional model equation was obtained. The program was run for several weeks on a single CPU and, at the time this paper was written, had not found the optimal solution, but several potential expressions had arisen from the process.

Most of the formulae that the application seems to gravitate to, revolve around an exponential model where the exponent term has an inverse time dependency. FORMULIZE was found to have the greatest difficulty with the high β curves where the deceleration of the meteor happens very abruptly over short time scales. It was found that running FORMULIZE on each β case separately allowed for quicker convergence to a generic model, and, then, that model had its coefficients fit as a function of β. The following model has generated the best fit thus far, but we must emphasize this is still a work in progress and the model presented should therefore not be construed as the final best answer.

4 Fit performance

By the time of the IMC in September 2012, the result obtained from FORMULIZE was a good representation of the position propagation for β values less than 3. In equations (3)–(11), the position parameterization is shown as a function of time and β, where each c_i coefficient is itself a polynomial function of β. Again, these values and formulae should not be taken as final. The residual plots of Figure 3 are shown up to $\beta = 4$ since the coefficient fits for c_i were only valid up to that value. So, we found

$$x(t) = t + \frac{c_1 + c_2 t}{c_3 + c_4 e^{\frac{c_5 + c_6 t}{c_7 + e^t} + c_8 t}}, \quad (3)$$

where

$$c_1 = +0.34087 + 2.6517\beta \quad (4)$$

$$c_2 = -0.50104 - 2.6621\beta + 0.89817\beta^2 - 0.097926\beta^3 \quad (5)$$

$$c_3 = +0.81913 + 4.2046\beta - 1.5428\beta^2 + 0.16662\beta^3 \quad (6)$$

$$c_4 = +19.548 + 109.15\beta - 41.522\beta^2 + 4.0013\beta^3 \quad (7)$$

$$c_5 = +179.04 + 425.61\beta - 65.534\beta^2 \quad (8)$$

$$c_6 = +28.46 - 77.083\beta + 9.1122\beta^2 \quad (9)$$

$$c_7 = +45.027 + 129.66\beta - 228.04\beta^2 + 223.16\beta^3 - 124.53\beta^4 + 39.119\beta^5 - 6.4418\beta^6 + 0.42978\beta^7 \quad (10)$$

$$c_8 = +0.40731 + 0.033279\beta \quad (11)$$

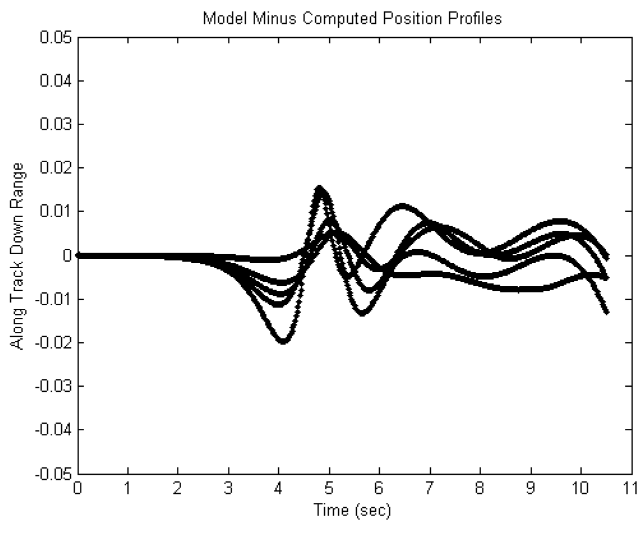

Figure 3 – Position residual (normalized units) versus time $x_{\text{model}}(t, \beta) - x(t, \beta)$ for various values of β ($\beta = 0.7551$, 1.5189, 2.2222, 2.9889, and 3.7965).

5 Conclusion and next steps

A preliminary model for meteor motion propagation has been obtained from the differential equations describing meteor ablation dynamics. This includes a position and velocity model as a function of time and mass loss parameter β. It currently is valid up to β values of 3 which represents about 80% of deeply penetrating fireballs encountered thus far. Further refinement of the model is a work in progress with the hope that a simpler expression can be obtained.

One issue is finding a model formula to handle the near discontinuity in the high β curves. One path may be to have two models that smoothly overlap the regimes of low β and high β. An attempt at finding a good model for high β is the goal for the next phase of the FORMULIZE runs. Another aspect that will be pursued is that the exponential integral function may need to be used as a formula within FORMULIZE. This is being investigated as a possibility. A third approach is to remove the terminal velocity portion of the position curves that FORMULIZE is trying to work on. They were put in to see if the long constant velocity tails could help bound the behavior of the functions fitting the decelerating sections of the curves. However, an alternative is to assume we only have to fit over the luminous flight portion of the track. Thus finding an expression that may be able to bend abruptly for high mass loss parameter but behaves poorly after the bend in the data may be adequate, because we would never be fitting the model to dark flight measurements. So long as the function behaves well during luminous flight measurements, the model should work seamlessly within the trajectory fitting application.

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