ISBN 978-2-87355-024-4

Proceedings of the International Meteor Conference
La Palma, Canary Islands, Spain
20–23 September, 2012

Published by the International Meteor Organization 2013
Edited by Marc Gyssens and Paul Roggemans

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Editing team and Organization

Publisher: The International Meteor Organization
Editors: Marc Gyssens and Paul Roggemans
Typesetting: LaTeX 2_ε (with styles from Imolate 2.4 by Chris Trayner)

Printed in Belgium

Legal address: International Meteor Organization, Mattheessensstraat 60, 2540 Hove, Belgium

Distribution

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A flexible fireball entry track calculation program

Esko Lyytinen[1] and Maria Gritsevich[1,2,3,4]

[1] **Finnish Fireball Working Group**
esko.lyytinen@jippii.fi

[2] **Finnish Geodetic Institute, Geodeetinrinne 2, P. O. Box 15, FIN-02431 Masala, Finland**
maria.gritsevich@fgi.fi

[3] **Institute of Mechanics and Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Michurinsky Pr. 1, Moscow, 119192, Russia**

[4] **Dorodnitsyn Computing Center, Russian Academy of Science, Vavilova Ul. 40, Moscow, 119333, Russia**

A computer program developed by Esko Lyytinen is currently in use for trajectory analysis of meteors collected by the Finnish Fireball Working Group. This paper provides instructions for its use along with processing examples from different phases of the calculation. The program is written in Microsoft Excel and it is available for download via the following link: http://lyytinen.name/esko/fb_entry_vers_1.zip.

1 Introduction

Trajectory analysis of meteors ablating in the Earth's atmosphere is a precursor to orbit parameters estimation and association with parent bodies in the Solar System. A spreadsheet program for trajectory estimation written for Microsoft Excel will be described by walking through several problem sets. The test examples have been generated from real fireball cases, but there is no assumption that the data are the latest checked best measurements available. The first example is based on a classical type deceleration model algorithm ("base model"), and we provide explanations and instructions for running this case. Later on, new examples are added, such as including a better deceleration model based on an approximate formula of the analytic solution of the meteor dynamic equations. It is however preferable to get familiar with the less advanced modeling method first, because the higher fidelity model is only applicable to some meteors. The advanced methodology is best applied to video data where each frame's direction is available from at least one station.

Besides the advanced deceleration treatment, there are also many special cases for which it will be useful to have special examples, e.g., single-station meteors, when special "tricks" must be used. An example are apparent fall-angle observations where only limited knowledge of the actual azimuth propagation is known. Furthermore, low-altitude angle observations from all-sky cameras may have the altitude calibration of poorer quality than the azimuth calibration. For simultaneously timed observational data (e.g., from video records), only the azimuth directions could be utilized in deriving solutions with other station data.

Such "tricks" will require the user to edit the Excel spreadsheet. Thus, experience using Microsoft Excel or a similar program is preferable. Even in normal meteor cases, new observation rows need to be generated (copied), to replace the old ones (from the base model), and the mutual timing data may need to be adjusted. Through direct manipulation of the spreadsheets, it is hoped that the user will obtain a good understanding of the principles of the program, instead of automatically following the procedures. The program includes essential algorithmic components explained in published papers and online resources (Ceplecha, 1984; Gural, 2012), while other resources are complementary. As an example, we refer to the spreadsheet List which allows one to calculate orbital elements from a meteor's radiant position and speed[1] and its associated publication (Langbroek, 2004).

Our first base model example is described in the following section. This example then is followed with the general functioning and principles of the program.

2 First example fireball 20120727 at 23:45:25 UT (yyyymmdd format)

This first case may be considered "ordinary" and was an expected small meteorite dropper. According to some visual observations, there was evidence for fragmentation, which cannot be seen in the video sum-image data. This was previously modeled to have the biggest fragment to be of 0.15 kg in mass and chondritic density of 3.5 g/cm^3.

There are data from four stations available, although of different accuracy. Thus the observations were weighted accordingly, and in our final solution, data from only two stations (Mikkeli and Joutsa) were effectively used. Now we will walk through the program steps.

[1] http://marcolangbroek.tripod.com/metsoft.html.

2.1 General lay-out of the program

To follow along, we invite the reader to open with MS Excel the spreadsheet fb_entry_20120727.xls.

At this time, everything may not be very logically situated due to historical reasons in the development. Cells colored red are calculated automatically according to given formulas and should therefore not be altered. The exception are the observation rows, which can be added for a new case by copying existing lines and/or deleted. In general, all colored shells (e.g., marked in violet) require caution. Each directional observation requires its own row.

Column O from rows 6 to 41 (in this example) with some free space in between contains the unknowns. The actual calculations are done in a Cartesian coordinate system that has the x, y plane tangent to the Earth (sea-level) surface, the tangent point being the origin of this x, y, z coordinate system. The Earth is modeled as an ellipsoid. Cells C2 and C3 contain the coordinates of the origin. The coordinates of the origin and observing site are given in decimal format of longitude and latitude, with East and North positive. Here, we are using WGS84 coordinates, but this is not critical if all processing and measurements are in the same system.

The shell A10 is one of the program's key shells, where the resulting value must be minimized by the Excel solver. In the sheet-region between C5 to J15, we have included some helpful calculations for the expected meteorite impact site. This is not essential to this sheet, however, because the full use of these calculations requires input from other programs dealing with ablation rate and dark flight simulation. The observing site coordinates are given in columns C and D, and the height above sea level (kilometers) in column B. Columns K and L contain the observed azimuth and altitude angle observations. The station locations are converted to the Cartesian x, y, z coordinate system (columns H, I, and J), and also the angular directions are transformed into this system as well (columns M and N). All x, y, and z values are given in kilometers. Column O has the timing of each observation, the values of which are dependent on the unknowns (shell O57 is linked to cell O12). The cells O58, O59, etc., are linked to their predecessors. In the Mikkeli data, every fifth video frame was measured from a sum-image (peak-hold image) that showed every fifth frame of the PAL video. The Joutsa data are similar.

Column P contains the time altered for deceleration calculations. The calculations are actually performed as if having a constant velocity (originally from cell O6) according to the "time" in column P. The linking of O and P values yields an approximation of the deceleration.

The unknowns O7, O8, and O9 define one point in the track. It is reasonable to fix one of these and look for the other two in the solution. In most instances, the value of O9 (the z coordinate) is set and fixed as zero. Thus, the values O7 and O8 (defined in kilometers) indicate where the continuation of the track would cross the ground plane. This approximates where the track would cross the Earth surface. The approximation is good if the cells O7 and O8 have relatively small values. If this is not the case, after finding the solution, one has to change the origin to (C2, D2) and solve the problem again. The calculated geographical coordinates of this crossing are placed in the cells P7 and P8. Cell O4 holds the time when this ground plane crossing would have occurred using the original velocity. This O4 value can be freely selected for a solution. One can adjust this value to ensure that all the timing values in the solution are nonnegative and do not have big numeric values. One can insert a value into cell J6 (taken from another program for dark flight simulation) that refers to how far this Earth-surface crossing is from the expected meteorite fall (in kilometers). Then, cells H6 and H7 will contain the approximation of the predicted meteorite fall location, without taking into account wind effects. (In the region between cells C12 and F14, we further calculated the wind-affected coordinates for the case when a small-size meteorite fall has been predicted. Again, the values in cells C12 and C14 (in kilometers) are input parameters provided by a complementary program.)

The geographical coordinates in P7, P8; H6, H7; and F12, F14 are calculated only approximately with scaled coordinate shifts. The same is true for columns BB and BC which contain the geographical coordinates of the individual observations. These are projected down along the normal onto the x, y plane (i.e., not projected down locally).

Thus, the treatment of original observations is accurate for an oblate Earth.

Columns AE, AF, and AG show the calculated Cartesian coordinates of each observation, which are converted to geographic longitude and latitude and given in columns BB and BC as described above.

Since z is not the actual height above sea-level, the actual height is better (and quite well) approximated at column AH with a correction applied to the AG value. Column AU contains individual weight values for each observation. Only the relative value for these is important, and we mostly use integer values. The value 100 is typically used for a good observation. Column AX contains the total error of each observation in degrees. Column AV has the weights multiplied by the squared errors. Note that this error is not given in degrees. Looking at the formulas in AV, AW, and AX, one can see the dependency between the values in column AT and the degree values. The cell A10 (the one which has to be minimized) has the sum of the values in column AV.

To summarize, the main principle of the program can be stated as follows.

Values of the unknowns (at column O) define the model-based direction for each observation. These are compared to the actual observation directions. And the (weighted) sum of the squares of the errors is minimized by means of the Excel solver. Once solved the result is a single entry track obtained from these observations.

The column `BI` has dimensionless velocity values, defined as the ratio of velocity to its entry value (given in cell `O6`). The column `BJ` shows velocity in kilometers. These values are the mean values between the previous sight-line and line in question (as seen in the calculation for cells in column `BI`). Column `BK` contains the absolute deceleration values (where applicable). This requires the successive rows (around the ones under consideration) to have the timing difference as constant, though corresponding formulae can be adjusted further when needed. The absolute deceleration values are estimated as a velocity difference (between the next and the current row) divided by the timing difference. Column `BL` has the height, as copied from column `AH`, to enable easy comparison of the velocity and deceleration with height.

As to the choice of deceleration model, this will be discussed in more detail later in the paper.

3 Experimenting with the program

It is possible to make one's own convergence tests with the solver. Please remember to save to a new file and not to the original spreadsheet in order not to accidentally over-write it.

Open the sheet. On the Excel *Tools* menu, one should find the *Solver*. The solver routine is a Microsoft Excel add-in program that is available when installing Microsoft Office or Excel. To use it in Excel, however, one must load it first following these 5 steps according to Microsoft Office's instructions:

1. Click the *File* tab, and then click *Options*.
2. Click *Add-Ins*, and then in the *Manage* box, select *Excel Add-ins*.
3. Click *Go*.
4. In the *Add-Ins available* box, select the *Solver Add-in* check box, and then click *OK*.
   (a) If *Solver Add-in* is not listed in the *Add-Ins available* box, click *Browse* to locate the add-in.
   (b) If prompted that the *Solver Add-in* is not currently installed, click *Yes* to install it.
5. When the *Solver Add-in* has been loaded, the *Solver* command is available in the *Analysis* group on the *Data* tab.

One can also get familiar with this function by looking up *Define and solve a problem by using Solver*, a section under the *Info* menu of Excel.

Now, one can select the solver by choosing *Solve*. The iteration process should soon stop (once it finds the best solution according to given criteria) and provide *Solver details*. Select *OK*, with the *Keep solver solution* option. Then do some minor editing in some of the cells containing unknowns in column `O`. Keep `O9` set to zero. If one decides to change the deceleration parameters in `O40` and `O41`, we suggest to make only relatively minor changes, especially to cell `O41`. Preferably do not increase the value of `O41` beyond 9. After these changes, run *Solve* again. Check the new result to see if it converged to the one which was originally found.

One can also try changing the origin (`C2`, `D2`). Again, make only a relatively small shift in this case. After solving for the shifted values, one can further shift the origin in the same direction. By making too big a shift in one move, one may "lose" the solution.

In a solution with different origin, the `O7` and `O8` values will have changed, but also the radiant directions will have changed a bit because of the different reference frame employed.

In this first case, the Mikkeli station was quite nearby, so one should not change the `D2` cell (origin latitude) by more than only a few tenths of a degree at a time.

One can also test the impact of changing the value of `O4` by a few seconds at a time.

Finally, one can also consider to alter the different observation weight values at column `AU`, and use one's "intuitive" solution to see how much it may differ (e.g., the impact site predictions) from the original solution.

4 The "old" deceleration model

As mentioned previously, the positions along the track are calculated as if the meteor would have constant (atmospheric entry) velocity relative to the adjusted time at column `P`. They are calculated from the corresponding values at column `O`.

The formula for the adjusted time at column `P` is a power-law expression with the power given (see the unknowns column) in cell `O41`. To avoid raising a negative value to a power, we first square the timing and then raise this positive number to the power `$O$41/2`. While in the final solution we should not be dealing with negative values, they may occur during the iteration process when the solver is searching for a solution. Our method to deal with this possibility prevents that the solver would stop with an error.

The position along the track follows this power-law function. Since velocity is the time derivative of the position, the corresponding power exponent for the velocity is one less than the power exponent for the position.

There is one more "hidden" parameter in the formulae corresponding to column `P` (which now has the value

"1" in ...+1) that defines how long before the first row (57) timing (in seconds) the power function starts at zero. The value used is not so critical, but it should be somewhat before the atmosphere begins to initiate any deceleration, e.g., just prior to the luminous trajectory segment. This value can be changed in the formulae, but then it must also be changed in every row (i.e., if changed in one line then copied to all other rows). For a different value of the timing offset, the power (in O41) also requires a different value. For fast, big-slope meteors, a small value (e.g., 1 second or even smaller) is advisable, but for slow and/or small-slope tracks, a value of a few seconds may yield better results.

According to this formula, the deceleration continually increases with time and typically with deeper penetration into the atmosphere. This scenario is well suited for the case of fully ablated meteoroids. For meteorite dropping cases, the deceleration reaches its peak within the luminous segment of the trajectory, and its absolute value typically decreases closer to the terminal velocity point. In this sheet, the deceleration can be modeled as constant close to the end of the luminous flight. However, this is only applicable if there are corresponding spreadsheet rows with constant relatively small time spacing. Notice that not all of these rows need to refer to an actual observation, as one can set the weight to zero. Assuming that there are evenly spaced rows (as said, preferably with small time spacing) close to the terminal fireball point, one can copy the formula from column P46 to some of the final cells in column P. In the example "base model" sheet, this formula has been copied to the last row in the Mikkeli observations.

Note that there is a requirement to have at least three rows above the one under consideration with the same time spacing. Again, copied rows not corresponding with actual observations are inserted with zero weight. In practice, such rows are only copied partially, such as row 64 in the example sheet.

If the constant deceleration formula is in use, this should be applied to all observing stations with timed data close to the end, and then have the constant velocity model to start from the end of known timing measurements, which can be seen only after an approximate solution is obtained.

If the close-to-terminal-point observations from some stations do not have timing information and only provide the track position, then there is no need to adjust a station with the alternate constant deceleration formula in column P. The timing of such observation is in any case unknown in column O (see the Pieksämäki observations both at row 74 and 75).

We advise it may be more reasonable to do this with respect to a separate ablation model program. If such a program is available, it should be used and the results then coupled together.

The coefficient of the magnitude of deceleration (cell O40) is evaluated from the solver solution. Therefore, it affects the value of cell O41 as well. If the observation data are sufficiently precise for a short registered trail, the value in O41 may not be derived sufficiently accurate. Thus, other parameters may be unreliable as well. To solve this problem, one may use a complementary ablation model. When the results of both models are compared, one may manually adjust the value of cell O41 (and decide on whether or not to copy the formula in cell P46) with new tested solver solutions to obtain an acceptable correspondence with the spreadsheet and the externally run ablation model. In many cases, however, it is expected that the value of cell O41 can be evaluated satisfactorily from the observational data. Note that even a relatively small change to O41 may require a big change to O40 in order to keep overall deceleration about the same.

It has been observed that if cell O41 has a value greater than about 9, the solver may fail to start converging. But if convergence has been achieved, this will probably continue for values in cell O41 greater than 9, if needed. In case of convergence problems, it typically helps to enter a value smaller than 9 after modification to allow convergence.

5 How to make the solution converge correctly from an initial guess

The next step is to make the model converge to the correct solution without a priori knowledge. (In case of having difficulties with this, the solutions are copied to column M. One should keep in mind, however, that if the origin (C2, D2) or cell O4 has been changed, the same solution will not apply.)

The procedure is to copy the entire original Excel sheet, and abandon the previous solution. Now readjust all the unknowns and resolve. One can proceed without further instructions, though the guidelines below may be helpful.

At first, set deceleration to zero (place "0" in cell O40; cell O41 can be left as it is or, more generally, set to another value, e.g., "7"). We attempt to get a very rough idea of where the meteor has fallen. In the example, it was seen to the south of Mikkeli. And as observed from Mikkeli, it was flying from right to left, making it roughly from west to east. Thus, enter "270" into cell O10. If the meteor made a steep entry (as was the case in the example), then use for O11 something in the range from 60° to 90°. If an estimate is not available, it is typically better to overesitmate the steepness than the other way around. Therefore, the input value of "60" is usually satisfactory. Regarding the velocity value in the cell O6, a range of 15 to 25 km/s is recommended for meteors assumed slow, 40 to 60 km/s for meteors assumed fast, and something in between for meteors assumed to have an intermediate velocity. Put "0" into O7 and O8. Put into the origin (C2, D2) a moderately good approximation. Now it is possible to try to converge optimally to a solution. The most reliable methodology is the following.

1. 1. Select the *Solver*, and solve now for only the timing unknowns, in this example, cells `O12` to `O16`. (To select such a group of unknowns, there is a special separator in EXCEL that may depend on the initial user's settings, but by default the correct expression would be `O12:O16`. Other separators (by default the semi-colon and the comma) are used between sets of unknown variables, if those in between are not to be used.

2. If Step 1 leads to a solution roughly looking reasonable, accept it and select *Solve* again. Now solve for only `O7` and `O8`.

   Repeat Steps 1 and 2 a few times. If the output values are reasonable (e.g., do not result in too large absolute values), one can copy this early solution into an independent column, for example, column `L`, if column `M` is already in use.

   If case the next step fails, one may start again from this saved solution without having to go back to Steps 1 and 2.

3. Now select both the cells `O7` and `O8`, and also the range `O12` to `O16`, and solve. If this still looks promising (the cell `A10` should not show a large value), save the result again to an independent column.

4. Add cell `O11` to the ones already selected at Step 3. (i.e., use the interval `O11:O16` instead of `O12:O16`). Again, save the obtained result if it looks reasonable. Note that one may keep only the latest or latest two values resulting from the most recent steps.

5. Include cell `O10` to the *Solver* inputs and, if acceptable, save the result.

6. Include the velocity given in `O6`.

7. Now try with all the unknowns, except the deceleration value in `O40`. Remember not to select `O9` which is left at zero.

8. If Step 7 was successful, save the result and include also cell `O40` among the input values.

9. One may now also select `O41` with all the others listed above. If needed, go back to the previous solution saved.

10. If the values in `O7` and `O8` are deemed too large, then smoothly alter the origin (`C2`,`D2`).

    If `O7` and/or `O8` show large values, the solution itself should be correct. But since the coordinates in `O7` and `O8` refer to the z, y plane crossing, the plane may be too high up from the actual sea level surface for a quality solution. And, thus, it may be desirable to alter the origin site because of this and obtain a new solution.

If the meteor path has a very small slope with respect to the horizon, the convergence methodology may not perform very well. And if it is a grazing meteor, whose track does not meet the Earth surface at all, then the above approach cannot even converge. In this case, one must abandon the strategy of keeping `O9` at zero, and set either `O7` or `O8` to zero instead. It may be preferable (but not necessary) to make a choice of this type of meteor crossing that fixed plane more perpendicularly than in the other alternative.

Otherwise, a similar approach is recommended to assure that the result will converge correctly.

The above instructions can easily be tested with the given example and/or the user's own examples. After some experimenting, this process normally does not take much time, though, surely, some cases are more difficult than others. Also, one may always try a finer stepping in parameters in order to succeed. This can be tried from the very beginning or at any of the numbered steps above. Since the results from previous successful steps are saved, one can run the failed run of *Solver* again by trying smaller steps.

6 Testing with single-station data

Let us look back more closely to the data reduction in the case of single-station data only. One can test it using the above example by including only one station's data, either Mikkeli or Joutsa, to obtain the solution. Define the weights at column `AU` as zero for the Siuntio and Mikkeli stations. Then also set to zero the Joutsa weights so that only the Mikkeli observations have none zero weights.

When invoking the *Solver*, deselect the velocity `O6` and let it be fixed. Some of the timing unknowns could also be deselected, but this is not necessary. Even if the velocity unknowns were left to be solved, a convergence would occur but this would yield just some arbitrary value.

One can also leave the deceleration (`O40` and `O41`) fixed as if it were already derived, and just test the trajectory solution. Keep `O41` fixed (as derived earlier) and let `O40` vary (among the others). This means that the magnitude of the deceleration is free to vary but the "shape" of the deceleration curve is fixed. Finally, let `O41` be variable. The result thus obtained differs from the originally derived one if more flexibility is given to the deceleration. In our testing, the azimuth direction can differ by about 30° in the worst case. Since the radiant is at about 70° altitude, this corresponds to not more than about 10° on the sky, however.

One of the above cases of flexibility for the deceleration parameters must be selected and then tested further by changing (by a few km/s, for example) the velocity `O6` (fix it in the solution). With different values in cell `O6`, this should provide a result with similar values for the direction parameters O10 and O11. For every case run, a beginning height value is calculated in cell AH57. Alternate solutions can be obtained with different values of O6, until one is found for which the beginning height of the luminous flight is consistent with expectation. This is then the solution from single-station data only.

One can do the same test with effectively only the Joutsa data by letting only these observations have non-zero weights. But now all rows in column P data are dependent on cell O57 and, it would be incorrect to let this value change freely. The most correct way to deal with this would be to put this dependency in the first row of Joutsa data. In order to avoid this modification, however, one can simply leave the cell O12 (which is copied to O57) fixed, or, even simpler, enter this existing fixed numeric value into the cell O57. In that case, remember to reinsert the reference =O12 into cell O57 later.

Similarly behaving results are expected from the Joutsa data. The Mikkeli data are more suitable to this single-station example, because the Mikkeli station was closer to the meteor, and also because it had a camera with a longer focal length. On the other hand, the fireball's video-images were more widely spread in the Mikkeli camera, except at the very beginning and end portions of the track.

When using single-station data only, it is still possible to test the quality of the solutions, without prior knowledge of the correct one. It should help if one sets the deceleration to zero in O40. After this has been iterated into a reasonable solution, let cell O41 be fixed (a modest value of around 4 to 8) and then allow O40 to be derived. Finally, one can test keeping O41 as an unknown. The insights obtained based on an ablation model (when available), could then lead to a more precise result (shown in cell O41) achieved with trial and error by comparing and altering. This is beneficial when the quality of the observations is not especially good with a sufficiently long, internally-timed observation track. One can say that the entry direction and deceleration both affect the apparent angular velocity; however, these effects should be derived separately. The deceleration is very small at the beginning, and the general geometry (directions, distances, and velocity) mostly influences the angular velocity behavior, while the deceleration is mostly effective near the end. This makes the task solvable, since the problem is partially separable. This requires observations to cover as much as possible of the luminous flight.

It may be of some help to have the (approximate) analytical formulae for velocity in single-station solutions. This may be a way to incorporate as good a deceleration model as possible, and such a model has been implemented in the example sheet referred to in the following section.

Note that, for high entry velocity meteors which completely ablate in the atmosphere, deceleration has less of an effect on the described solution, and these single-station cases are easier if only a sufficiently long track (e.g., from a grazer) is available.

7 The advanced deceleration model

A sample sheet with the case of fireball 20120509 corresponds to an expected small meteorite dropper, and it incorporates the approximate formula from the analytical solution of the entry differential equations. For more details, see the paper by Gritsevich (2009). For easy reference, the formula used is also visible in the sheet as a screen capture image from that paper.

In this equation, the term $0.83\beta(1-v)$ is of lesser importance, and if the numeric value of this term was known, the value of v could be analytically solved for (here, the dimensionless velocity v is defined as the ratio of the actual meteor velocity V to its pre-entry velocity value V_{e}). In this selected example case, an acceptable solution could be reached in only one step by getting the approximate value of v from the previous iteration. Alternatively, one could manually set $v = 1$ (i.e., actual velocity equals pre-entry velocity) for the beginning segment of the luminous trajectory, and choose a smaller value of v for the later flight-stage (as a first approximation to start the iterative method). In the Excel spreadsheet, the value "1" has been manually input into cell BI7 and the value "0.55" into cell BI51 for this purpose. The program integrates the position of the meteoroid over time incorporating the dimensionless velocity value v from the formula mentioned above. Also, the integration involves the "altered time" in column P. The numerical integrations in this example are introduced to use the time step according to the camera's frame rate. Note that, here, the cameras have different frame rates of 25 and 30 with corresponding steps of 0.04 s and 0.033333 s.

The integration method uses the simplest approach, with the velocity for a given step calculated from the previous observation height, etc. More elaborate methods could be introduced, but might make the spreadsheet more "burdensome" for computation. The *Solver* may in some cases need more than a thousand iterations. Considering that the observations contain measurement errors, and that the formula is an approximation, the selected integration method is expected to be sufficient. Also, some simple trials to improve the method may easily lead to a circular reference preventing the calculations to run further.

If a different time step is used for different observing site data, then these integrations may not give exactly the same results and negatively impact the final solution. It is therefore desirable not to have a big time difference in steps during observations. In general, it is better to keep the step small, even though there were no observational data for each step (corresponding to a row of its own in the spreadsheet), as with the Mikkeli site data. We assume that using both 0.04 and 0.0333 seconds in the same spreadsheet is acceptable. Bigger time-step differences have been tried for other examples, with no noticeable harm to the solution.

From this method, one obtains values for the dynamic equations parameters α and β. The estimations of entry mass and end mass calculated from these can be seen in cells G19 and J19. The entry mass estimation is calculated by means of the α value and the entry radiant altitude ("slope") value with some other parameters affecting the final result, such as the meteorite density (given in cell F19) and parameters in cells E19 and I19 of the spreadsheet. These input parameters luckily only change over a relatively small range. The parameter α (the ballistic coefficient) characterizes the aero-braking efficiency and serves as initial dimensionless meteoroid mass definition. The parameter is proportional to the ratio of the mass of the atmospheric column along the trajectory to the meteoroid initial mass. The mass loss parameter β serves as an initial dimensionless meteoroid energy. This parameter is proportional to the ratio of the initial kinetic energy to the energy which is required to completely destroy the meteoroid body during its atmospheric entry. Revealing these parameters based on observations is outside the scope of these instructions, though. More details on this matter are given by Gritsevich (2009).

There are different formulae applied to save for further use in the cells P36, P37, and P38. The formula in cell P38 needs to be copied for this model's use into each row with this type of integration, except the first one, for every case. In the first row of every integration section, one should copy the formula in P36, which is the one used in our "old" deceleration model. Alternatively, one can input a direct copy using the same row from column O. If the old deceleration parameter (cell O30) is fixed to zero, the old type formula makes a direct copy of the cell in column O to column P for the row under consideration, as suggested above.

If one has solved the problem with the old type deceleration parameters, these can be used in the new model, in the first row of each internally timed sequence (copy formula form cell P37) to get a value in column O (in the first row under consideration) linked to the column P value at the same row. As to the correctness of the actual solution, there could be a direct copy formula from O to P, but if having used the old deceleration formula (with valid parameters), then this ascertains that the values in column O for different stations will be in the same actual time scale. If one does not have these old parameters, but wants to have this kind of time synchronization, then it is advised first to solve the case according to the old model.

If there are no active dependencies of the old type model remaining, except those linking the start of each segment and/or some individual observations, as is the case here in cells O46 to P46, then the end result of the track and its associated parameters, such as α and β, are not affected.

If the old-type formulas and parameters remain more or less in use, another problem may arise. Applying the power deceleration formula to time values that are too big could reverse the direction of motion. It may happen that the solver finds such a time value and further tries to converge around this. This may lead to unrealistic solutions. Such a situation may also be encountered when the old deceleration model alone is in use. In such cases, the convergence typically gets much slower and in certain situations may not find a good minimum (convergence may be slow for other reasons as well). If this is suspected, check the (unknown) timing values at column O. Edit that value which is found to be considerably bigger than a reasonable estimate for the flight profile in question. Then try to solve again.

The advantage of using the based-on-old-type model parameters, as in this example, is that the timing data in column O for each observation and station (within some uncertainties) is in the very same "time system", and hence they are directly comparable to each other.

It should be noted that, in many cases with poorer quality data, the parameter β may not be very well estimated. This is the case in this worked example. You can notice this by trying to find solutions with the weights of Mikkeli and Pukinmäki changed relative to each other. In this case, the difficulty is expected to arise because the Pukinmäki site was quite distant from the meteor and did not even observe the meteor to as low an altitude as it was observed in Mikkeli. With Pukinmäki, automatic image-pixel-coordinates for each frame are in use. In the Mikkeli observations, the measurements were made from a (peak-hold) sum image that was made to capture only every fifth frame, in order to be able to get internally timed manual measurements. This meteor came towards Mikkeli with slow angular velocity and so these frames (of the very bright meteor) cannot be individually measured for most of the flight. The internally timed measurements were taken only close to the end.

Surely, a more definitive case could have been used as the example, but we found it beneficial to demonstrate what kind of difficulties could arise in actual calculations.

At this point, one could also take the resulting tabulated velocity and height values from the derived solution with the advanced model (as well as only from our base model) to be used in the algorithm by Gritsevich (2009), that includes the more accurate solutions based on the dynamic differential equations.

Note. The implemented deceleration model involves the scale height parameter in order to analytically describe the isothermal atmospheric profile with the density decreasing exponentially with the height. Especially in high northern latitudes like Finland during the winter, the polar vortex makes the stratospheric constant pressure surfaces to occur at considerably lower heights than they would normally appear at in the exponential atmosphere. The model can in principle be corrected for this, at least if actual stratospheric data are available. This has already been tested with the `fb_entry` program, but we plan to study this in more detail, present results, and release an `fb_entry` program which account for such corrections later.

8 How to use only the internally-timed azimuth values from the camera data

The reader should refer to the EXCEL spreadsheet for this example, `fb_entry_20120727_examples.xls`.

It is typically found for all-sky video cameras that the low elevation angles are not as well calibrated as the azimuth angles. In such cases, it is possible to rely strictly on the azimuth angles. One requires another complementary camera to provide a more complete set of directional data. Also, the azimuth values of the all-sky camera under consideration need to be internally timestamped. For the track derivation, it is also important that the data from the complementary camera be internally timed. Otherwise, the case can be analyzed only for the retrieval of the meteor velocity.

This has been applied to several fireball cases, in Finland, Russia, and the USA. However, these spreadsheets are too complicated (also for the referred Finnish "Lammi case"). The presented case, therefore, is a slight modification of the 2012 July 27 case for the Siuntio station, using its (only) two internally timed data-points. The spreadsheet that has already been in use with these instructions has also another modified version, which is `fb_entry_20120727_examples.xls`. We have in this same spreadsheet another special example that we will refer to later.

The Siuntio observations at rows `70` and `71` with the time spacing of 1.96 s (as seen in cell `O71`) have been consequently altered for this purpose. Regarding the formulas, only the `AV` column cells (`AV70` and `AV71`) have been altered.

Instead of the total error, there is the difference between the observed (directions transformed according to the frame) azimuth from column `BF` and the modeled value. Because the usual data errors are (practically speaking) in radians, this total error is also roughly converted to radians. This allows the weighting values to be comparable to the more normal data azimuth-elevation values.

The column `AX` displays the total error which is due to the difference of the azimuth values at columns `BF` and `AZ`. One can see that, after fitting, the real error in these observations is about 4° and the azimuth error discrepancy below $0^{\circ}\!.2$.

Since this camera was calibrated a long time ago, the azimuth values are actually not very reliable. Therefore, this has been presented here as an example only, illustrating how to work with data like these. The apparent azimuth errors are not necessarily the real errors in the azimuth. They are only based on the best fit. Indeed, the key practical idea is not to get a good triangulation of where the fireball track is, but to have the angular velocity well-fit to the other observations, as such may tell, for example, on the actual entry direction in azimuth and/or on the situation in between two stations with observations from opposite sides of the track.

One can obtain the best results using a full sequence of internally timed video frames. Also, it is desirable to have the azimuth values quite well calibrated, hopefully better than in this example.

It may happen that the azimuth value is near 180 degrees. The allowed values for difference subtraction here are between $-180°$ and $+180°$. One value can be close to $-180°$ and the other one approaching $+180°$. Then the difference that arises may be close to 0° or may be around 360°, preventing the convergence to the correct solution. If this happens, one may for example treat the difference by using the principle value of the function `DEGREES(ASIN(SIN(RADIANS("difference"))))`. This is converted to radians by dividing by $180/\pi \approx 57.3$, but this last step can be omitted if the first operation in the formula referred above (`DEGREES`) is omitted as well.

9 An additional property to deal with apparent fall-angle observations

The reader should refer to the EXCEL spreadsheet for this example, which is `fb_entry_20120727_examples`, row (approximately) `107`.

This property is mainly useful if the spreadsheet is used to analyze visual observations only or observations available from only a single camera supported by several visual observations. This of course can be used with camera observations, but the method uses some approximations, and in some cases may not provide all the information that another method described later, may give.

In this approach, the azimuth direction of the observation need not be known. Some value for the elevation angle will be needed, however. If the observed angle is steep, and the meteor is observed not far above the horizon, the exact elevation is not so important (at the accuracy level of visual observations). It will not produce much difference if one has used the value of 10° for the elevation while it was actually only 5°, for example.

If the flight is more horizontal, then the elevation angle is much more important. Quite typically this is only very approximately known from visual observations, but it can be helpful. If so, then another method is recommended. This other method is especially recommended if it looks highly probable that the observation is near the radiant. In that case the elevation angle will not be needed, but will (hopefully) be derived from other observations. This "additional property" method calculates what these fall angles would be relative to the horizon from the observation and correspondingly from the track fit, and makes a comparison. This method also has a weakness if, for example, one makes an observation at an elevation of 45° with this angle as 80°. Alternatively, an observation also at an elevation of 45° with this angle as 100° will be converted to the same value relative to the horizon.

These situations may be a little bit difficult to visualize, but this method is expected to be best suited with steeply sloped angle observations. If practically all observed angles are steep, then the entry itself is likely to be relatively steep too.

This approach is quite simple to use. As for other observations, the observing site coordinates are placed into columns `B`, `C`, and `D`. The observed fall angle is entered at `K`, and the corresponding altitude of this observation at column `L`.

The weight value to each observation is given in the cells at column `AU`. After all (or sufficiently many) observations, directions, and fall angles are included, one tries to find a solution in the usual way. After the fit, the error in every fall-angle observation (as calculated at the horizon) is shown at column `AI`, and also copied to column `AX`.

The measure of the error is practically the same as in the azimuth-elevation direction observations, therefore similar weighting values can be used if they correspond to similar accuracy observations.

10 Apparent fall-angle near the radiant

The reader should refer to the Excel spreadsheet for this example, which is `fb_entry_20120727_examples_B`, rows `112–113`.

This method is of use mainly with expected meteorite droppers when only visual observations are available. If there exists a fall-angle observation near where the direct track would have intercepted the Earth's surface, this can be of significant importance in the derivation of where the intercept point happened and consequently yield the whole trajectory.

This method also assumes that there are other observations that at least approximately can define the radiant direction. If so, then this special observation need not require anything else other than the apparent fall angle relative to the horizon or vertical.

This observation is constructed by means of two directional measurements. The first has the azimuth-elevation direction totally unknown, and the second is adjusted relative to the first so that their mutual situation is according to the observed fall angle (given here in cell `J111`). Elaborate spherical trigonometry formulae for this are probably not needed, because this is expected to be applied to visual observations for the most part.

In general, it helps before starting with this method that a rough solution already exists. Given a rough estimate, the first observation timing (cell `P111`) can be small enough (strongly negative value) referring to the highest point (which is allowed to be much higher than the real luminous heights) and close to the actual radiant direction, as seen from the observer site. For the second observation, the timing is kept as unknown. In practice, it does not affect the solution if the resulting height (that corresponds to these observations) from the ground level is far from the real height of the meteoroid. The angle can still be considered the same. If, however, the observation has a reliable value for the observed track length, then one may try to adjust the timing data by trial and error, so that the heights (above the ground in kilometers) are close to what one may assume. If the modeled track length is not what has been observed, then a determination can be made based on whether the track's Earth's surface crossing is closer to or more distant from the observer than the modeled result.

11 Apparent fall angle, more general case

The reader should refer to the Excel spreadsheet for this example, which is `fb_entry_20120727_examples_B`, rows `125–126`.

In principle, this is formulated in a similar way to the previous section. In this case however, the altitude is fixed and the azimuth is unknown (`K125` with reference to `O20` in this case). The timings of both observation lines are unknowns (`P125` and `P126` with references to `O21` and `O22`).

Especially with a horizontally moving meteor, it may occur that the azimuth angle is quite well known, but not the elevation angle. Then the selection of which of these parameters is to be taken as unknown can be altered. If both are reliably known, then it may be better to have the observations as (two different) normal azimuth-elevation observations. If there are enough reliable fall-angle observations available ("surrounding" the low height entry path), it is possible that they will define the whole track approximately, even without any known azimuth directions. In this spreadsheet, the artificial fall-angle observations were generated mostly close to the trajectory's end point. One may test a case of eliminating all the dual measurement azimuth-elevation data by setting these rows with zero weight at column `AU` from rows `57` to `92`, and then trying to solve. In this case, we suggest to leave the velocity (cell `O6`) outside of the unknowns range, because it cannot be derived. It does converge to some (maybe questionable) value, even if not left out. Now the track appears to be shifted from the actual one by some tens of kilometers. The direction of entry, however, is acceptable.

Here, the fall-angle values were rounded to a reasonable visual accuracy level (actually these may still be more accurate than typically with visual observations).

If there are originally no azimuth directions, then it might be difficult to "find" the roughly correct track from which a solution would later converge to. We have not faced such a situation in practice, but we have calculated a case where we could test the use of only fall-angle values, which led to a nearly correct track.

12 Modeling the Earth's gravitation during the flight

The reader should refer to the EXCEL spreadsheet for this example, which is `fb_entry_20120727_examples_C`.

Most fireballs do not need to be corrected for the effects of gravity. Only very low velocity and low radiant-angle fireballs may require correction for gravitation. So, if there is no discernible need, it may be preferable to assume the gravitational effect to be insignificant. In Finland, there have been a few cases in which this has been applicable. In particular, for the 2012 October 21 British fireball it was found necessary to have the entire flight covered by a single fit.

If one is not sure about including the effects of gravity, a decision can be made based on testing if the inclusion of this effect decreases the residuals of the observations or not.

There are at least three reasons mentioned below, as to why one should try to avoid this. It has been typical in Finland to start a new fireball case from an earlier `fb_entry` spreadsheet and modify it accordingly. Especially if there had already been observations taken from the same stations, then the coordinates can be used again.

First of all, when one takes the Earth's gravitational effect into account, one should keep that in mind if altering the model further to study a future (no gravity) case. Otherwise, more harm could come from using this option than any advantage provided.

The second concern is that in order to have this effect well incorporated, it would actually require a numeric integration or some other elaborate program. In the current spreadsheet, the effect is just added at the coordinate level. The atmospheric deceleration is calculated along the track and the deceleration should also "damp" the accumulated gravity effect. The additional downwards velocity only increases with time. Due to this the correction for the gravitation effect, one could easily overcorrect with a decelerating meteoroid.

In practice, the effect is typically included only in the upper part of the trajectory.

In a meteor grazer geometry, like the mentioned British grazer, the gravitational effect can probably be applied to the whole track, because it is acting orthogonal to the atmospheric deceleration.

The third reason to avoid including this effect is the following. The radiant altitude in cell `O11` is for the straight track, even if the effect is included. This is the same (for the straight and curved legs) at the tangent point, but if the origin is not also in this location, or very close to it, it becomes more or less indefinite as to where the cell value `O11` actually refers to, if one wants to be accurate.

In the example sheet, this effect is revealed in the column `AG` cells which are colored in gray. There is the addition of `-0.0049*(O63-O$63)^2` (the number in `O63` varies according to the row). This addition is valid in this sheet for a height of 46.7 km or above in the different station observations. Geometrically, this curves the track relative to the mentioned height, but the track continues as a straight line below this level. At this "point" the straight track is tangent to the curved track.

While adjusting the `O63` cell, one should look at the heights at column `AH` to make sure that the gravitation formulae are applied at the same height range for every station.

In this example sheet, the gravitational effect is not at all needed, but was included to show how it works. The origin was also changed (compared to the other examples within this case) close to the tangent point, because of the "third reason" explained above.

13 Estimating the accuracy of the result

This is not easy to do with any method. The spreadsheet program does not compute any statistical error estimates. This may be considered a weakness, but there are reasons why such error values may be highly misleading. Before going to the actual means used for estimating the errors, let us discuss the problems associated with error estimation in more detail.

Video observations provide, in principle, looking directions to the meteor for each frame. The UFOANALYZER pogram can provide that for each half-frame by taking advantage of the interleaved nature of most commercial video systems. Thus from a 25 fps PAL video we have 50 directions for each second of visible meteor track, and we have had cases with more than 200 directions from one station. This results in a set of problems that we had to face in the spreadsheet program. Assume, for example, that we have 200 observations (and possibly more) from one station and from a second station only a fixed image from which we measure a few track points. If we assume that both the direction measurements of each frame from the first station and the few individually measured track points from the second station are of equal accuracy, and consequently apply the same weight for each, this will most likely lead to a highly erroneous solution. The internally timed observations can more or less define the true entry direction, and if there are a lot of them, the single-station data solution will be dominant. The individual systematic errors resulting from image calibration and/or enlarged meteor images in those 200 data points can be considerably larger than what statistics would tell us should be required for final accuracy (due to the law of large numbers). The result may thus be that the fitted track considerably deviates from the second station's non-timed track, even though the measurements of this second site's measurements are in reality good.

In such a situation, the few non-timed track positions must be weighted much more heavily than each of the video-frame sequence data points. The weight values themselves are subjective estimates, which are not directly obtained by examining possible individual errors.

Although we have emphasized the usefulness of fitting to internally timed observations, this method has its own disadvantages if not applied with sufficient care.

Essentially systematic errors complicate the error analysis. If we would use a locally linearized least squares fit with statistical error values, the error values cannot be trusted, because the systematic portion of the observation errors are not mutually independent. Indeed, if one applies random errors to each point of the data and generates a number of solutions, this can lead to incorrect error analysis. In the baseline solution, there will be observations with systematic errors and these errors will not get removed by adding additional random errors. The solution will remain biased. Of course, if sufficiently large random errors are added, the error estimates will be large enough to cover the correct solution as well, but the required random errors may significantly exceed the true errors of each observation. In asteroid and comet orbit analysis, such a random error generation has been used successfully, because their observations are probably more mutually independent.

One practical means of estimating the errors is by generating pairwise or coupled solutions when there are more than two stations of data, and compare them.

Other methods involve approximate calculations of how much the assumed errors (in angle space) are effective at the distance of the fireball, and then also take into account the angle between the observation planes (not available directly from our Excel sheet). This may lead to some idea on the accuracy of the luminous entry path derivation. The errors in determining a meteorite's final entry location goes beyond the scope of this paper. Typically, these are much larger than the errors in the position of the luminous flight track.

One may also couple the observations by using only a fraction or subset of the available data and then try to solve. Unfortunately, this does not help much to avoid the systematic errors.

It is useful to doublecheck each station's observations, to see if they deviate from a straight line (great circle on the sky) by looking at their residuals with respect to the fit trajectory. If one of the stations has a long internally timed track, then it may be worthwhile to try a single-station solution. This may provide some information on the systematic errors along the track, which do not show up in the straight line check.

If one is interested in trying to determine a parent body by comparing the derived meteoroid's solar system orbit with comet or asteroid orbits, then the semi-major axis accuracy value is of major importance. Typically, this is strongly dependent on the accuracy of the entry velocity value. To check the accuracy of this parameter, we have kept the entry track fixed at the final common solution and tried to get the velocity derived individually from each station's data, where applicable.

14 Miscellaneous items

14.1 What to do, if the solver does not converge

After one has already obtained a good solution and then proceeds to make a minor change to the sheet, it may occur that the program does not converge anymore but stops immediately, even though the differences between the solution and the measurements are not fully minimized. To help solve this problem, one should try to make more significant changes to some quantity. It is also recommended to slightly change the velocity value in cell `O6`, by 1% for example. It typically helps the convergence. If there is a relatively large value in the deceleration cell `O41` (more than 9, for example), this may be one cause for the problem. Decreasing it to a value below 9 typically helps, and, after convergence, cell `O41` will have its final value resulting from the fit, even though it can be more than 9.

Selecting the *Use automatic scaling* setting to be on in the *Solver Options* may sometimes help, but at other times (especially if some timing happens to be nearly zero), this may even prevent any further convergence. In most cases there is not much harm with this "not able to converge properly". In the *Solver Options*, it is worth to keep very small values in *Precision*, *Tolerance*, and *Convergence*. In the *Derivates*, there are two options: *Forward* and *Central*. Sometimes the *Forward* option seems to work more efficiently and more easily leads to convergence. In some instances, however, better convergence can be achieved with the *Central* option.

14.2 How to check for accurate ground track location

The reader should refer to Excel spreadsheet for this example, `fb_entry_20120727_examples_D`, row `112`.

This is of real importance for at least low entry angle meteorite droppers.

The geographic coordinates in P7:P8 and H6:H7, as well as for the track points in columns BB and BC, are only calculated approximately by scaling the latitude and longitude coordinate intervals. Even the curvature of the equal-value latitude track is not taken into account in this approximation. With steep entry (and origin near the fall site) these typically are sufficiently accurate, but, otherwise, especially with a big east-west span of viewing coverage, this may not be accurate enough.

Here, we describe how to check one point at a time, to see how close it is and if it is possible to correct the values.

One could test if the continuation of the track passes directly overhead. For this, a row with a normal observation type is added. Here, we have an elevation angle of 90°, and thus the azimuth direction can take on any value. The timing is an unknown, now preferably in column P, not resulting in any harm from the deceleration model that might have "stopped" the meteoroid before this point. One of the coordinates is a given value and the other is solved as an unknown to have a reference point exactly on the calculated ground track.

This method can also be applied in a slightly modified way to calculate a previously fixed point, such as the luminous flight end point ground coordinates. To do this, one has to copy and paste an existing observation row which has just this lowest point into free open rows in the spreadsheet. Next, one must copy the numeric value from the same row (by using the '*Paste special/Values* option) into the new rows's column P. Keep the weight for this row small, but above zero. Copy the rough coordinates from columns BB and BC to columns C and D of the new row and insert the elevation angle of 90° at column L. Then solve with the *Solver* for cells C and D of this row. One should then obtain the accurate geographical coordinates of the ground point instead of the approximate ones in columns BB and BC.

14.3 How to deal with calculated dark flight length (by another program)

As mentioned before, one can use a value of how much before the Earth surface crossing that the fall happened, and place it into cell J6 having the modeled fall location (without wind effects) in H6:H7. This is sufficient for steep atmospheric entries, but is not adequate for low-angle entries. Even if one has this value from some other program, the value may be large and very sensitive to small differences between this program and the other source. This is susceptible to error, if, for example, the steepness values are actually not in the same spot. Moreover, one has to note that the origin needs to be selected so that the values in O7 and O8 are small, because these values refer to the point where the straight track is crossing the $z = 0$ plane in the Cartesian x, y, z coordinates. This plane is tangent to the Earth's surface at the origin, and not the sea level surface, and a low-angle entry case may therefore produce errors.

It may be typical that the true end flight length is shorter than the value for the fall site before the plane crossing in the J6 cell. So, it may be preferable to use the true dark flight length estimate obtained with a complementary program instead.

With the accuracy typically required for this, it is expected to be sufficient if this has been calculated by means of the coordinates in H6 and H7 and those found at columns BB and BC for the observation that corresponds to the end of the luminous flight (if one exists).

Spherical trigonometry can be used, but it is expected that rougher calculation is as accurate as a dark flight model. One can assume the latitude difference for 1° to be 111 km and the longitude difference for 1° to be (111 km) $\times \cos\varphi$, with φ the latitude, and then use the Pythagorean Theorem to calculate the dark flight (ground path) length in kilometers.

If the cells H6 and H7 values do not show the desired length (the one obtained with a complementary program), one should adjust the value of cell J6 by trial and error to get improved results.

For an Earth grazer, the Earth's surface crossing value does not exist, of course. Interestingly, the dark flight may nevertheless result in a value of finite length. Thus, some other means to calculate the corresponding point along the track will be needed. One can check some accurate track points and do spherical trigonometry to continue with trial and error searching for this case.

14.4 Other possible uses of the program

Assume that you see from your camera image a distant light near the horizon, such as, for example, a TV-station mast or a tower. If one has accurate geographical coordinates of the station and the light of the mast, one can use the program to calculate the azimuth direction to this mast.

One must choose the origin coordinates to be exactly those of the station, and include an observation row with the coordinates of the mast in columns C and D (B is not important in this case). Then, one must calculate the direction by means of the mast's x and y coordinates, found in columns H and I, for example, in the sheet with the formula DEGREES(ATAN2(In/Hn)), where n is the row number under consideration. One can also calculate the direct distance with the coordinates, by means of the Theorem of Pythagoras. And if distances along the surfaces are needed, an approximate correction to this is easy to calculate. For example, the formula 2*6370*ASIN(d_dist/(2*6370)) will provide a good value up to distances of a few thousand kilometers. In this case, d_dist has been calculated geometrically from the coordinates, as above.

The program has also been successfully used in the calculation of the location and height of thunderstorm TLE's, such as sprites.

14.5 Possible Excel errors

Since the beginning of the development of this program, the following Excel error has been noticed, except in the latest version of MS Office Excel 2003 SP3, where another error has appeared. After some use of the *Solver* with quite extensive models, the *Solver* stops working. Nothing happens when trying to solve; the *Solver* sheet does not even become visible. Alternatively, the *Solver* does not show a visible result. In most instances, ending the Excel session and starting a new one helps. However, it may be that after a few restarts of the Excel, even this may not help, and one has to restart the computer to get it working again.

If the *Solver* stops working, one should save the existing sheet before exiting Excel. In this case, save the program into a file with a new (duplicate) name.

We have learned how to effectively avoid this error. When selecting the *Solver* and then selecting the input cells for solving, one can make a shift to another originally non-visible position on the sheet, but one should avoid such shifting at this stage. If one cannot select the cells from what is visible, it is better to try to edit these manually. Even having one such cell and a quick Solve run with stop and *Keep Solver solution* should assure the *Solver* to appear in the right visible corner of the sheet during its next use, and one can then select visually the needed unknowns.

The mentioned error in the 2003 version may be even more critical.

After getting used to the *Solver*, and beginning to edit some cell content, it is beneficial to remember the following. This mentioned 2003 version could crash without allowing you to save your edited sheet. Therefore, it is recommended to save frequently, especially in the case of significant editing changes. It is better to edit the program only after starting Excel and loading the sheet, but before starting to run the *Solver*.

15 Conclusions

The key features of the flexible fireball entry track calculation program used for reduction analysis of the Finnish Fireball Working Group meteors has been presented. The authors hope that the reader finds these instructions to be helpful and constructive. The program has been officially released for public use and made available in particular through the internet.[2] Program requests and relevant comments can also be addressed directly to the authors. Some real meteor examples have been presented to get the interested reader started, with a focus on special issues such as gravity inclusion and error propagation, which are not widely taken into account in meteor studies elsewhere. We have also included within the paper a description of some common issues such as ensuring faster solutions and intermediate technical problems the authors have already faced while performing data reductions of meteor observations.

Acknowledgements

The authors would like to warmly thank Paul Roggemans and Marc Gyssens for the enormous efforts they spent on getting the 2012 IMC Proceedings issue out in a timely fashion and of the highest quality possible. The authors would like to thank to Peter Gural who provided valuable comments and edited an earlier version of the manuscript. The authors are also very grateful to the other members of the Finnish Fireball Working Group for their enthusiasm, thorough help with collecting meteor observations, and other relevant tasks.

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[2] `http://lyytinen.name/esko/fb_entry_vers_1.zip`.